FREE CONVECTION AROUND A PLATE WITH TEMPERATURE DECREASING EXPONENTIALLY ALONG THE SURFACE

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Results of the numerical solution of a system describing nonselfsimilar natural convection in a laminar boundary layer near a vertical plate with exponentially decreasing temperature distribution are discussed. The nature of the development of the thermal and dynamical boundary layers as well as the friction stress and heat flux along the plate surface is given.

Let us consider free-convective motion along a vertical plate in a fluid with constant physical properties. The fundamental system of equations hence takes the following form [1]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\vartheta, \tag{1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

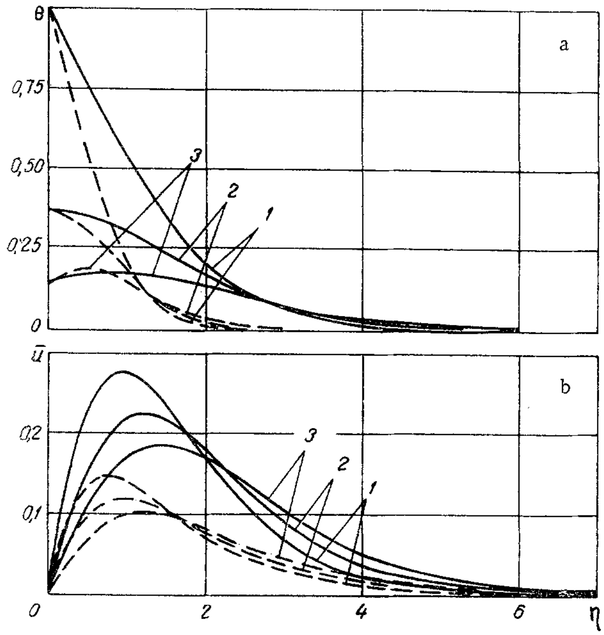

Fig. 1

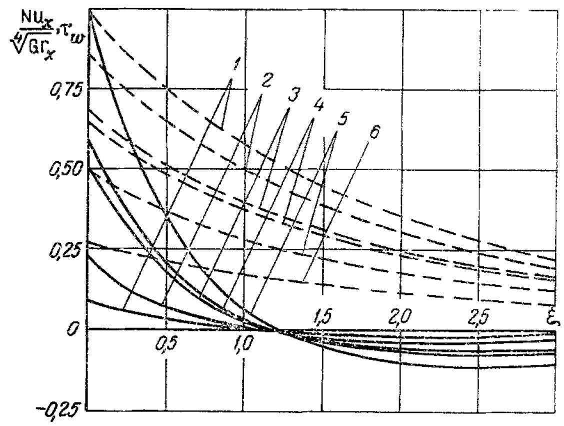

Fig. 2

Fig. 1. Temperature profiles in the boundary layer a and velocity profiles in the boundary layer b; solid lines correspond to the number Pr = 0.7, and the dashes to Pr = 5.0: 1) $\xi = 0$; 2) 1; 3) 2.

Fig. 2. Friction stress and heat flux distribution on a plate surface. Solid lines) heat flux $Nu_x/\sqrt[4]{Gr_x}$. dashed friction stress $\overline{\tau_w}$: 1) Pr = 0.01; 2) 0.1; 3) 0.7; 4) 1; 5) 5; 6) 50.

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$$u\frac{\partial\vartheta}{\partial x}+v\frac{\partial\vartheta}{\partial y}=a\frac{\partial^2\vartheta}{\partial y^2} \tag{3}$$

with the boundary conditions

$$\begin{aligned} &u=0,\ v=0,\ \vartheta=\vartheta_{st} \quad \text{for } y=0,\\ &u=0,\ \vartheta=0 \quad \text{for } y\to\infty. \end{aligned} \tag{4}$$

Selfsimilar solutions which are possible for a power-law or exponential temperature distribution on the plate surface [2] are usually considered in computing the free convection along a plate. An attempt is made in [3] at a combined taking account of the heat conduction and free convection in assigning a definite law of rib outline variation. The solution for free convection will be selfsimilar when the rib is outlined specially. A number of papers devoted to nonselfsimilar solutions in the boundary layer is based on almost selfsimilar methods of solution [4]. The results elucidated there are of limited value since they are applicable for a relatively small value of the expansion parameter.

It has been established in an experimental investigation of heat conduction in elements of an armature which are thin vertical plates with Bi $\ll$ 1 heated from below, that the temperature distribution can be represented as

$$\vartheta_{st}=\vartheta_0\exp(-mx). \tag{5}$$

To solve the boundary-value problem, let us introduce the transformation

$$\begin{aligned} &\xi=mx,\ \eta=cy/x^{1/4},\ c=\left(g\beta\frac{t_0-t_\infty}{4\nu^2}\right)^{1/4},\\ &\psi(x,\ y)=4\nu cx^{3/4}f(\xi,\ \eta),\ \Theta=\frac{t-t_\infty}{t_0-t_\infty}, \end{aligned} \tag{6}$$

where $\psi(x, y)$ is the stream function satisfying the continuity equation (2).

The system (1)-(3) in the new variables reduces to the following:

$$\frac{\partial^3 f}{\partial\eta^3}+3f\frac{\partial^2 f}{\partial\eta^2}-2\left(\frac{\partial f}{\partial\eta}\right)^2+\Theta=4\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial\xi}-\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial\xi}\right), \tag{7}$$

$$\frac{1}{\Pr}\frac{\partial^2\Theta}{\partial\eta^2}+3f\frac{\partial\Theta}{\partial\eta}=4\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\Theta}{\partial\xi}-\frac{\partial f}{\partial\xi}\frac{\partial\Theta}{\partial\eta}\right) \tag{8}$$

with the boundary conditions

$$\begin{aligned} &f=0,\ \frac{\partial f}{\partial\eta}=0,\ \Theta=\exp(-\xi) \quad \text{for } \eta=0,\\ &\frac{\partial f}{\partial\eta}=0,\ \Theta=0 \quad \text{for } \eta\to\infty. \end{aligned} \tag{9}$$

Selfsimilar solutions exist for a homogeneous temperature profile (m = 0) since the right sides of (7) and (8) vanish here. Therefore, the difficulties of assigning a profile at the initial point of integration can be bypassed. Moreover, the equations presented are universal for any exponential temperature profile on the wall and it is sufficient to solve it once.

An implicit difference scheme was used to solve (7) and (8). The derivatives with respect to η in the equations were replaced by central differences. A difference approximation was used for the derivative with respect to ξ. Consequently, the initial equation was reduced to a system of nonlinear algebraic equations which was solved by iterations. A linear system was hence solved in each iteration by using the known factorization method.

The typical nature of the thermal and hydrodynamical boundary layer development is represented in Fig. 1a, b. Shown in Fig. 1a is the development of the thermal boundary layer starting from an initial selfsimilar profile. The essential singularity is that the temperature maximum is displaced within the thermal boundary layer starting with some cross section. Such a shift is explained by an upward rise in the hotter layers and a diminution in the plate temperature with height. The results of numerical computations for a dimensionless velocity profile $u = u/4\nu c^2x^{1/2}$ are presented in Fig. 1b. It can be noted that the thickness of the dynamical boundary layer grows as ξ increases, and the maximum value of the velocity decreases and is shifted to the right.

It is seen from Fig. 2 that the dimensionless friction stress on the wall $\overline{\tau}_w = \tau_w/4\nu\mu c^3 x^{1/4}$ decreases. As has been mentioned earlier, the heat flux is first directed from the wall and then starting with some ξ from the boundary layer to the wall, as is explained by the change in sign of $Nu_x/\sqrt[4]{Gr_x}$.

For a high-temperature armature this heating by fluid layers rising upward is not desirable. The mentioned computation affords the possibility of establishing the optimum height dimension of an armature.

NOTATION

u, v	are the projections of the velocity vector on the x, y axes, respectively;
x	is the longitudinal coordinate;
y	is the transverse coordinate;
ν	is the coefficient of kinematic viscosity;
μ	is the coefficient of dynamic viscosity;
g	is the acceleration of gravity;
β	is the coefficient of volume expansion;
α_x	is the local heat exchange coefficient;
λ	is the coefficient of fluid heat conduction;
λ_T	is the coefficient of heat conduction of the plate;
t	is the temperature;
$\vartheta = t - t_\infty$	is the excess temperature;
ξ, η	are the selfsimilar variable longitudinal and transverse coordinates, respectively;
f, Θ	are the dimensionless stream function and temperature;
$Pr = \nu/a$	is the Prandtl number;
$Nu_x = (\alpha_x x/\lambda)\Theta_{st}$	is the local Nusselt number;
$Gr_x = g\beta\vartheta_0 x^3/4\nu^2$	is the local Grausshopf number;
$Bi = \alpha\delta/\lambda_T$	is the Biot criterion;
$\overline{u} = u/4\nu c^2 x^{1/2}$	is the dimensionless velocity;
$\overline{\tau}_w = \tau_w/4\nu\mu c^3 x^{1/4}$	is the dimensionless friction stress;
δ	is the plate thickness.

Subscripts

w	is the wall;
x	is the local value dependent on the coordinate;
0	is the origin;
∞	is the external flow.

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